Transient heat transfer

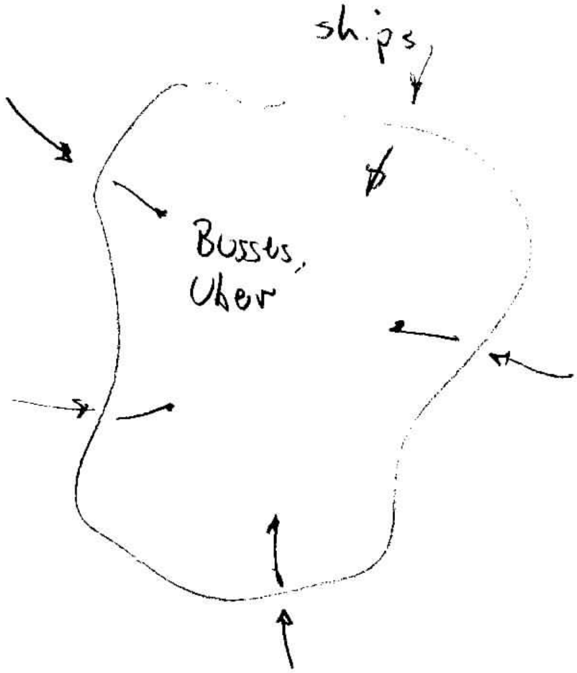
lumped (no spatial var.) ODEs

spatial variation (in all directions?) PDEs

Cruise ships on island problem

Goal: - avoid too many people in ports

- distribute people as uniformily as possible over island.



How to do this?
what infrastructure do you need?

Now at end of the day, get people out of off the island as soon as possible.

Now think heat.

$\underline{\underline{\text{If}}}$ body is at uniform temperature (∞ fast q transfer)

uniform $T(t)$ k c ρ $\rightarrow h, T_\infty$

Considering body as a whole...

rate of change of thermal energy	=	rate of heat loss to surroundings

$$\frac{d}{dt}(mcT) = -hA_{surf}(T_s - T_\infty)$$

Assume $m = \rho V$

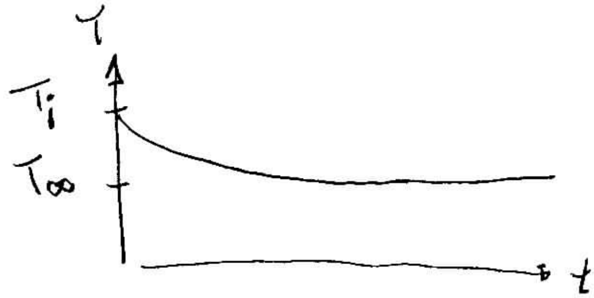
$$\rho V c \frac{dT}{dt} = -hA_s(T_s - T_\infty)$$

Why?

$$\frac{dT}{dt} = -\left(\frac{hA_s}{\rho V c}\right)(T - T_\infty)$$

this is $T(x)$ if uniform temp. in body !!! Why is this ok?

We expect (if object is hot and fluid is cooler)



I.C. $t = 0$; $T = T_{init}$

With no harm, let $\boxed{\theta = T - T_\infty}$

so I.C. $t = 0$; $\theta = T_i - T_\infty \equiv \theta_i$

and ODE is $\dfrac{d\theta}{dt} = -\left(\dfrac{hA_s}{\rho V c}\right)\theta = -b\theta$ $\quad b \equiv \dfrac{hA_s}{\rho V c}$

$[b] = 1/\text{time}$

$$\frac{d\theta}{\theta} = -b\,dt$$

$$\ln|\theta| = -bt + \not{c}$$ Drop | | on $|\theta|$?

@ $t=0$

$$\ln \theta_i = \not{c}$$

$$\ln\theta = -bt + \ln\theta_i$$

$$\ln\frac{\theta}{\theta_i} = -bt \quad \text{or} \quad \frac{\theta}{\theta_i} = e^{-bt}$$

$$\boxed{\frac{T - T_\infty}{T_i - T_\infty} = e^{-bt}}$$

Go one step further...

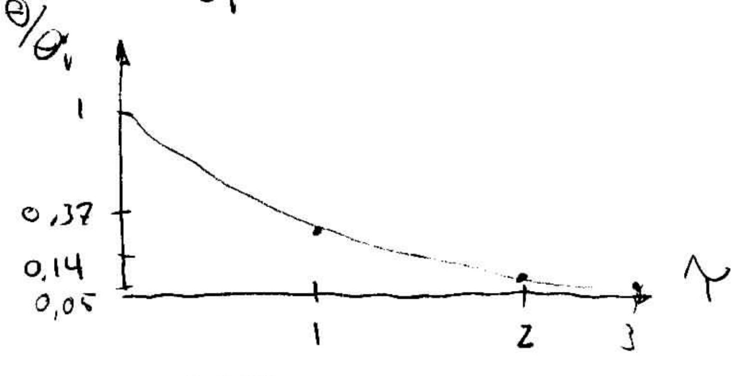
$$\tau = bt \quad \text{(dimensionless)}$$
$$= \left(\frac{1}{\text{time constant}}\right) t$$

call

$$\boxed{t_{char} = 1/b}$$ characteristic time scale

$$\tau = t/t_{char}$$

finally $$\frac{\theta}{\theta_i} = e^{-\tau} = e^{-t/t_{char}}$$



The current rate of heat transfer from the body to the surroundings is thus

$$\dot{Q} = hA_s\left(T_{(t)} - T_\infty\right) \qquad [W]$$

The total amount of heat transferred from body to surroundings from $t=0$ to arbitrary time t is then

$$Q = mC\left(T_{(t)} - T_i\right) \qquad [kJ, J]$$

Max. heat transferred to surroundings is ($t \to \infty$)

$$Q_{total} = mC\left(T_\infty - T_i\right) \qquad [kJ]$$

When is lumped analysis valid?

Really good conduction that exceeds convection

Could look at

$$\frac{\text{Convection to surface}}{\text{Conduction within body}} \sim \frac{h\,\Delta T}{\left(\frac{k}{L_c}\right)\Delta T} = \frac{hL_c}{k}$$

Char. length of body

$$\boxed{L_c \equiv \frac{V}{A_s}}$$

Called the Biot #

$$\boxed{Bi = \frac{hL_c}{k}}$$

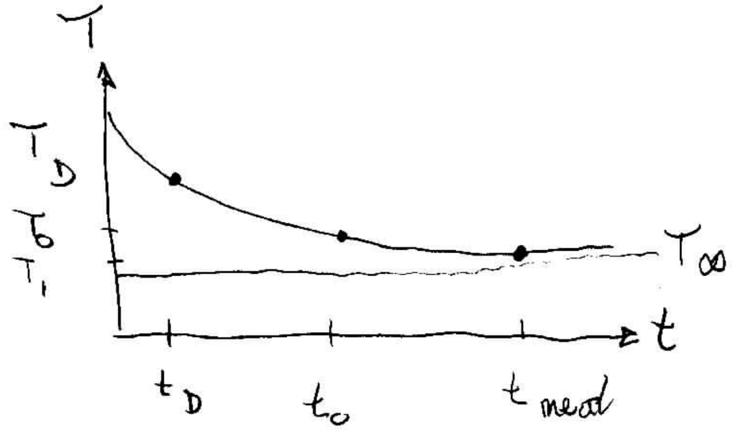
, + $Bi = 0$ lumped is exact! (∞ good cond.)

In reality $Bi < 0.1$ lumped is good to within about 5%

Ex1 T.V. shows ... time of death. (saved by the bell!)

At $t=0$ you find a dead body @ $T=T_0$

But death was earlier



T_∞ surrounding temp

T_D is known (98.6F)
t_D is unknown (what was it?)

$\Theta_D = T_D - T_\infty$
$\Theta_0 = T_0 - T_\infty$

T_0 is measured by you
$t_0 = 0$ (start the clock)

T_1 second T measurement $\quad \Theta_1 = T_1 - T_\infty$
$t_1 = \ldots$ pick a time

Our general solution for $Bi < 0.1$ is

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{-bt} \quad \text{or} \quad \Theta = \Theta_0 e^{-bt}$$

But we don't know b. So @ $t=t_1$ take $T=T_1$

or $\Theta = \Theta_1 = T_1 - T_\infty$

then $\Theta\big|_{t=t_1} = \Theta_1 = \Theta_0 e^{-bt_1} \Rightarrow e^{-bt_1} = \dfrac{\Theta_1}{\Theta_0}$

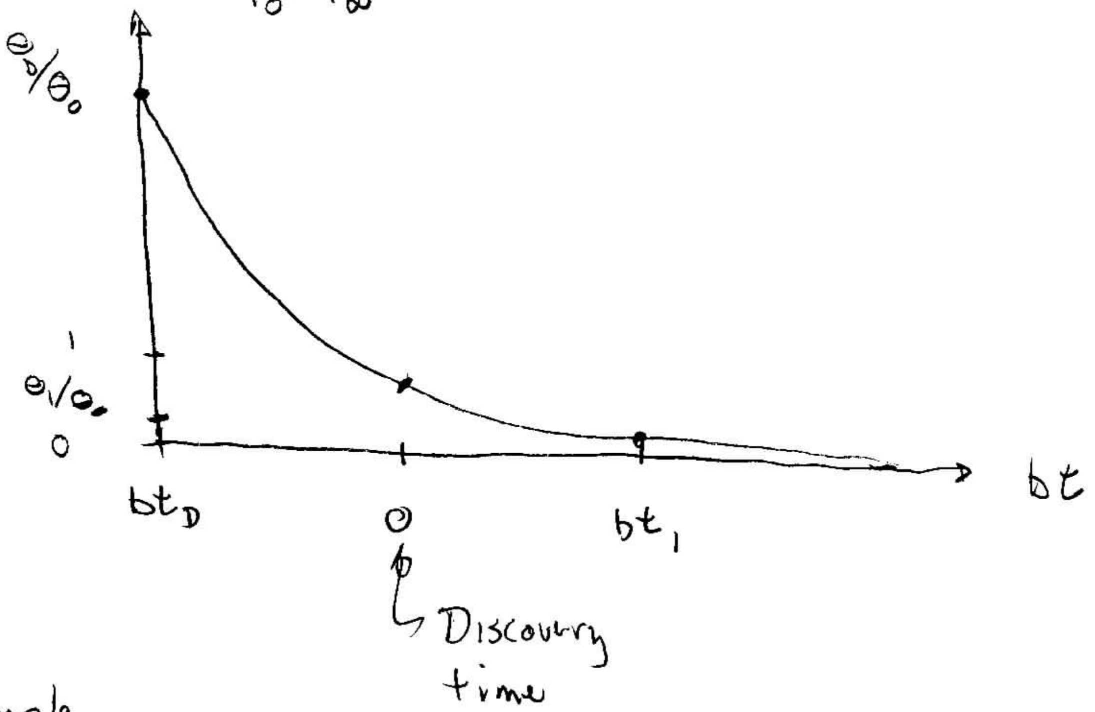
solve for $b = \dfrac{-1}{t_1} \ln\left(\dfrac{\Theta_1}{\Theta_0}\right) = \dfrac{-1}{t_1} \ln\left(\dfrac{T_1 - T_\infty}{T_0 - T_\infty}\right)$

you can now find t_D since

$$\theta\Big|_{t=t_D} = \Theta_D = \Theta_0 e^{-bt_D} \quad \text{or} \quad e^{-bt_D} = \Theta_D/\Theta_0$$

so $$t_D = \frac{-1}{b}\ln\left(\frac{\Theta_D}{\Theta_0}\right) = \frac{-1}{b}\ln\left(\frac{T_D - T_\infty}{T_0 - T_\infty}\right)$$

Finally... $\Theta/\Theta_0 = \dfrac{T - T_\infty}{T_0 - T_\infty}$



For example

S'pose $T_0 = 85\,°F$ @ $t_0 = 0$ hours $\qquad T_{amb} = T_\infty = 68\,°F$

wait 2 hours $T_1 = 74\,°F$ @ $t_1 = 2$ hours

then $$b = \left(\frac{-1}{2\,hrs}\right)\ln\left(\frac{74-68}{85-68}\right) \approx 0.5207\ hr^{-1}$$

so $$t_D = \left(\frac{-1}{0.5207}\ hr\right)\ln\left(\frac{98.6-68}{85-68}\right) \approx -1.129\ hr$$

or 1 hr 8 min after death

Ex

Use T-couple to measure gas T

$k = 35 \ W/mK$ $\quad \rho = 8500 \ kg/m^3$

$C_p = 320 \ \frac{J}{kg\,K}$ $\quad h = 210 \ W/m^2K$

- Want to measure within 99% of true $T_{fluid} = T_\infty$
- Neglect radiation! Always true?

For a sphere $L_{char} = L_c = \frac{\forall}{A_s} = \frac{\frac{1}{6}\pi D^3}{\pi D^2} = 1.67 \times 10^{-4} \ m$

Check out $Bi = \frac{h L_c}{k} = 0.001 < 0.1$ so lumped analysis is ok

To get within 99% of true gas temp

we want $\frac{T(t) - T_\infty}{T_i - T_\infty} = 0.01$

$b = \frac{h A_s}{\rho C_p \forall} = \frac{h}{\rho C_p L_c} = 0.462 \ sec^{-1}$

Put into $\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$

$0.01 = e^{-bt} \Rightarrow t \approx 10 \ sec$

Insert T-couple into fluid, wait 10 sec, read temp

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